LEARN: ENTANGLEMENT - Advanced

"I would not call entanglement ***one*** *but rather* ***the*** *characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought."* - Erwin Schrödinger

If there's a property that represents the essence itself of Quantum Physics (QP), compelling us to go beyond our conventional perception of the world, the choice is one by all odds: quantum entanglement.

This extraordinary feature of QP comes into play when several quantum systems are combined or, equivalently, when we consider composite quantum objects, which are composed of many (interacting) quantum parts. According to the **fourth postulate of quantum mechanics**, when an isolated physical system S is composed of two subsystems A and B, with Hilbert spaces $\mathscr{H}_A$ and $\mathscr{H}_B$, respectively, the Hilbert space of the total system S is $\mathscr{H}_S = \mathscr{H}_A \otimes \mathscr{H}_B$, where the symbol $\otimes$ identifies the tensor product. The states of the total system are all the vectors $|\Psi_S\rangle$ normalised to 1, which means $\langle\Psi_S|\Psi_S\rangle = 1$, contained in $\mathscr{H}_S$.

Combining two quantum systems opens up new scenarios. Sometimes it's possible to "recognise" the state of each subsystem A and B, because they are not combined together. Such states are called "separable" and take the form $|\Psi_S\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$, with $|\Psi_A\rangle$ in $\mathscr{H}_A$ such that $\langle\Psi_A|\Psi_A\rangle = 1$, and $|\Psi_B\rangle$ in $\mathscr{H}_B$ such that $\langle\Psi_B|\Psi_B\rangle = 1$. These are not the only allowed states for the total system S. In fact, due to the mathematical structure of the tensor product, there exist joint states $|\Psi_S\rangle$ for which the subsystems A and B become so knotted that they loose their individual identity. Mathematically, this means that they ***cannot*** be written in the form $|\Psi_S\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$. Such states are said to be "entangled" and the property related to such feature is named "entanglement".

Entanglement is an exquisitely quantum correlation created by interactions between subsystems. Moreover, since it is an intrinsically quantum property, there can be entanglement only if both subsystems are quantum. Let us consider the simplest multipartite quantum system S, namely, a system composed of two qubits: A and B. Among all the possible states of the total system S, there are special entangled states, named Bell states, which are *maximally* entangled because the amount of information that you can get on the second qubit B by performing a measurement on the first qubit A is maximal. The Bell states are:

$$|e_1\rangle = \frac{1}{\sqrt{2}}(|1_A\rangle \otimes |1_B\rangle + |0_A\rangle \otimes |0_B\rangle) \qquad |e_3\rangle = \frac{1}{\sqrt{2}}(|1_A\rangle \otimes |0_B\rangle + |0_A\rangle \otimes |1_B\rangle)$$

$$|e_2\rangle = \frac{1}{\sqrt{2}}(|1_A\rangle \otimes |1_B\rangle - |0_A\rangle \otimes |0_B\rangle) \qquad |e_4\rangle = \frac{1}{\sqrt{2}}(|1_A\rangle \otimes |0_B\rangle - |0_A\rangle \otimes |1_B\rangle)$$

Looking at these states, we cannot say in which state each subsystem is, for instance if A is in the state $|0\rangle$, $|1\rangle$ or a superposition of them. But we can determine the state of B after performing a measurement on A: for example, if the total system S is in the state $|e_1\rangle$ and we get 1 when measuring A, we know that B is in the state $|1\rangle$, whereas if measuring A we get 0, we know that B is in the state $|0\rangle$. Lastly, notice that entanglement is different from superposition: you need a compound system, made of two subsystems at least, in order to have entanglement. This is not required for quantum superpositions. Namely, entanglement stems from the superposition principle applied to multipartite systems. However, notice that, for instance, the state

$|\Psi_S\rangle = \frac{1}{\sqrt{2}}(|1_A\rangle \otimes |0_B\rangle + |1_A\rangle \otimes |1_B\rangle)$ is not entangled. We can indeed rewrite it as $|\Psi_S\rangle = \frac{1}{\sqrt{2}}|1_A\rangle \otimes (|0_B\rangle + |1_B\rangle)$, which is clearly separable, where it's easy to "recognise" the state of the subsystems A and B.

In words, due to entanglement, microscopic particles can become so strongly correlated that, loosely speaking, we say that they do not have anymore an "individual identity". In a way, many fascinating phenomena of Quantum Physics, but also most of the revolutionary Quantum Technologies, rely on entanglement. The idea of entanglement was first introduced in 1935 in a famous paper on the so-called EPR paradox, named after the three authors Albert Einstein, Boris Podolsky and Nathan Rose. The term "entanglement" was instead coined by Schrödinger, who, even if recognising its importance, was never satisfied by its consequences. One of the biggest problems was that entanglement seemed to violate the speed limit imposed by Special Relativity in the transmission of information. For instance, if we consider two entangled electrons, the result of the measurement that we perform on one of them will match the random result of the measurement of the other one, regardless of whether they are in the same lab or at the opposite far sides of the universe. Specifically, Einstein and his co-authors showed that the quantum formalism permitted the existence of certain two-particle states for which the correlations are so strong that one can predict either the momentum or the position of one by measuring the other, even when the two particles are widely separated and no longer interact. This posed a problem. The authors concluded that either information travels faster than light or that both momentum and position are well-defined all along, in contradiction with the Heisenberg uncertainty principle.

For several decades, a large number of scientists tried to search for alternatives to this apparent paradox, including Albert Einstein, who refused to accept what for him represented "a spooky action at a distance". Of course, faster-than-light communication could not be accepted as an option, so Einstein and his collaborators analysed the strong correlations between entangled particles in the context of a local physical reality. One solution, strenuously defended by Einstein, was that QP was not a complete theory, but that there were some predetermined *local-hidden variables* determining the results of measurements. The randomness in the outcomes would only be a manifestation of our ignorance, which would disappear once we discovered the more general theory which appropriately took into account the hidden variables. The dispute between the Einstein's faction and the quantum defenders seemed to be unsolvable, until a brilliant Irish physicist, John Bell, formulated in 1964 a theorem according to which, if Einstein was right, all experiments would have to satisfy certain inequalities, named indeed Bell inequalities, concerning correlations between two quantum systems. On the contrary, if even only one experimental observation violating such relations was obtained, this would rule out any possibility of local hidden variables.

Although the first pioneering attempts to demonstrate experimentally the violation of Bell inequalities date back to the 70s, the first clear results were not obtained until the 80s. Since then, a large number of experiments have shown that Bell inequalities are not satisfied and therefore Quantum Physics has been repeatedly empirically confirmed. "The experimental tests of Bell inequalities gave an unambiguous answer: entanglement cannot be understood as usual correlations, whose interpretation relies on the existence of common properties, originating in a common preparation, and remaining attached to each individual object after separation, as components of their physical reality. Quantum theory describes a pair of entangled objects as a single global quantum system, impossible to be thought of as two individual objects, even if the two components are far apart."[1]

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[1] Alain Aspect, Introduction to: J. S. Bell. "Speakable and Unspeakable in Quantum Mechanics: Collected Papers on Quantum Philosophy" - we recommend to look at it if you are interested in the topic.