LEARN: QUANTUM PHYSICS - Advanced

Quantum Physics was born more than 100 years ago, right when some physicists thought there wasn't much work left for them to do. In fact, at the end of the XIX century, Physics seemed to have reached its peak: the Maxwell equations, explaining electromagnetism and classical optics in a very beautiful and concise way, completed the Newtonian universe. From the XVII century onwards, the theoretical progress had enabled a chain of astonishing technological advancements. However, studying apparently minor bottlenecks in optics, physicists found that to explain some of the observed phenomena they needed to introduce a very weird object: a "quantum" of energy. This was the first attack to the tranquillity of Newton's world. Everything was about to change.

Quantum Physics arrived unexpectedly, following a contorted road, not really desired by anyone. Yet, it brought along huge innovation leading to a change in paradigm perhaps more radical than any other in the history of human thought. Quantum Physics is not just a theory: it is a new way of looking at the world. It forces us to abandon our comfort zone, go beyond our conventional mind frames, and open up to new things. This is also the reason why your first encounter with Quantum Physics can feel a bit harsh: relax, it is totally fine! Also the founders of the theory needed to build a completely new conceptual world in the attempt to understand those phenomena so strikingly different from what, until then, was considered common sense. Such a process naturally creates an internal struggle because of the contrast with the perception of the world that we start to form since our childhood. This is why learning about Quantum Physics is, beyond everything, a fantastic way to stimulate our creativity, our problem solving abilities, and our innovation attitude, since it forces us to think outside the box.

Through the years physicists managed to identify the minimal set of assumptions or axioms on which Quantum Physics is based; they are formulated in terms of certain mathematical concepts, and mastering their profound meaning requires some intellectual effort. It's not unusual that only after many years spent learning the theory, one fully grasps their consequences. However, they represent the core of the whole quantum world, a sort of basic grammar to start composing wonderful novels. Before introducing them, we need to make some considerations.

First of all, the number of postulates of Quantum Physics is various: they are often presented as three, sometimes as four, sometimes even as six. If you like to reduce to the essential minimum, you would need two (the state and the measurement ones) from which you can then derive all the others. We think that neither the proliferation nor the extreme reduction are a good way to introduce them to people that meet Quantum Physics for the first time, so here we choose the number that, in our opinion, best suits the needed logical steps for "first-time-encounters" with QP.

1. The **first postulate of quantum mechanics** defines the **states** of any isolated physical system: given an isolated quantum system, we associate to it a Hilbert space $\mathscr{H}$. Each physical state of the system is described by a normalised vector of $\mathscr{H}$, that is represented as $|\Psi\rangle$, such that $\langle\Psi|\Psi\rangle = 1$. The opposite is also true: each normalised element of $\mathscr{H}$ represents a possible state of a physical system.

2. The **second postulate of quantum mechanics** defines the **time evolution** of physical systems: the state of the system at a certain instant t_1, $|\Psi(t_1)\rangle$, is given by the state of the system at a certain instant t_0, $|\Psi(t_0)\rangle$, transformed by a unitary operator $\hat{U}$, namely $|\Psi(t_1)\rangle = \hat{U}_{t_1,t_0}|\Psi(t_0)\rangle$ with $\hat{U}^\dagger\hat{U} = \hat{\mathbb{1}}$.

In short, the evolution of physical systems is described by unitary operators[1].

3. The **third postulate of quantum mechanics** (minimal interpretation) is about **measurement**: the probability of obtaining the result m_ℓ performing a measurement of the observable $\hat{M}$, when the system is in the state $|\Psi\rangle$, is $p^{M}_{|\Psi\rangle}(m_\ell) = \langle\Psi|\ell\rangle\langle\ell|\Psi\rangle = |\langle\ell|\Psi\rangle|^2$, where $|\ell\rangle$ is the eigenstate of $\hat{M}$ with eigenvalue m_ℓ.

4. The **fourth postulate of quantum mechanics** defines **multipartite systems**: when an isolated physical system S is composed of two subsystems A and B, with Hilbert spaces $\mathscr{H}_A$ and $\mathscr{H}_B$, respectively, the Hilbert space of the total system S is $\mathscr{H}_S = \mathscr{H}_A \otimes \mathscr{H}_B$, where the symbol $\otimes$ identifies the tensor product.

[1] Adding to the second postulate the hypotheses of time being homogeneous and continuous, one can derive the Schrödinger equation.